

Econ 802

Second Midterm Exam

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All questions have equal weight. If something is unclear, please ask. In all questions you should assume that prices and income are strictly positive.

1. Define the utility function $u(x_1, x_2)$ to be the smallest integer that is greater than or equal to $x_1 + x_2$, where $x_1 \geq 0$ and $x_2 \geq 0$.
 - (a) Draw a graph illustrating some indifference sets and upper contour sets. Explain your reasoning in words.
 - (b) Do the preferences represented by this utility function satisfy weak monotonicity? Strong monotonicity? Strict convexity? Explain briefly in each case.
 - (c) Does this consumer's utility maximization problem always, sometimes, or never have a solution? Carefully justify your answer.

2. Consider the utility function $u(x_1, x_2) = x_1^{1/2} + x_2$. For the following questions, don't bother with Kuhn-Tucker multipliers. If necessary, just consider (p, m) values for which the demands are strictly positive.
 - (a) Derive the Marshallian demand functions and the indirect utility function.
 - (b) Derive the Hicksian demand functions and the expenditure function.
 - (c) Something unusual happens when you compare the Marshallian and Hicksian demand functions for good 1. Give a good verbal explanation.

3. For each of the following questions, make your answer as complete as possible.
 - (a) An Econ 201 student says: "The concept of utility maximization is too general to restrict consumer behavior in any observable way." Use the Slutsky equation to show mathematically why the student is incorrect, and justify your answer.
 - (b) An Econ 201 student says: "It doesn't make sense that if the wage goes up, labor supply could go down. The substitution effect should cause people to work more when wages increase." Explain the source of the student's confusion.

- (c) An Econ 201 student says: "I don't see how economists can draw indifference curves with one good on the horizontal axis and 'all other goods' on the vertical axis. If there are many goods, there should be many axes." Explain how you could justify using only two axes by imposing some restrictions on the prices.
4. A firm has the short run cost function $c(y, K) = rK + wy^{1/\beta}K^{-\alpha/\beta}$ where $r > 0$ is the price of capital, $w > 0$ is the price of labor, $K > 0$ is the stock of capital (fixed in the short run), and y is output. Assume $\alpha > 0$, $\beta > 0$, $\alpha + \beta = 1$, and $\beta < 1/2$.
- (a) Draw a graph showing the shapes of the curves for average fixed cost, average variable cost, average total cost, and marginal cost. Justify your answers.
- (b) Suppose there are n firms of this kind. They are identical except that they may have different capital stocks $K_i > 0$ for $i = 1 \dots n$. Explain mathematically how you would derive the short run market supply curve and show it on a graph.
- (c) In the long run the number of firms remains fixed at n , but each firm can vary its capital stock. Describe the long run market supply curve and show it on a graph.
5. Suppose there are identical consumers $i = 1 \dots n$ with utility functions $u_i(x_i, y_i) = ax_i - bx_i^2 + y_i$ and identical firms $j = 1 \dots m$ with cost functions $c_j(z_j) = gz_j + hz_j^2$, where z_j is the amount of the x good produced by j and $c_j(z_j)$ is the amount of the y good used by j . Each consumer has an identical endowment (w) of the y good. The price of the y good is equal to one. Assume $a > g > 0$, $b > 0$, and $h > 0$.
- (a) Solve mathematically for the equilibrium price p^* in the market for the x good. Then draw a graph showing the market demand and supply curves. Using your graph, explain what happens if the number of consumers (n) increases while the number of firms is held constant, and what happens if the number of firms (m) increases while the number of consumers is held constant.
- (b) Show that the equilibrium allocation of the x good and the y good from part (a) maximizes the sum of the utilities of the consumers. How would you interpret this result using a graph like the one in part (a)?
- (c) Suppose the government tells consumers that they must pay a tax of t per unit on each unit of the x good they buy. (Notice that it is the consumers, not the firms, who pay the tax.) All other assumptions are the same as before. Use a graph to explain what price consumers pay in equilibrium, what price firms receive, and what happens to total output. How does this affect your answer from part (b)?